

Sample of an assignment for students. (You can of course construct a number of other interesting investigations to suit your students and your course.)

For each of the following answer the result of the expression involving reversal matrices. Some answers require a matrix (M), others require a verbal description (V) of the type of matrix. Write the expression and answer on a sheet of paper. (Don't say it is a n by n matrix as an answer.)

1. $\mathbf{J}_4^T =$ _____ (M)

2. $\mathbf{J}_4^T =$ is a _____ type of matrix. (V)

3. $\mathbf{J}_4^2 =$ is _____ (M)

4. $\mathbf{J}_4^{-1} =$ is a _____ type of matrix. (V)

5. $\mathbf{J}_4^T * \mathbf{J}_4 =$ is _____ (M)

6. $\mathbf{J}_4 =$ is a _____ type of matrix. (V) Hint : use answer to 5

7. $\mathbf{J}_4 + \mathbf{I}_4 =$ is _____ (M)

8. Let $\mathbf{W} = \mathbf{J}_4 + \mathbf{I}_4$. Find $\text{rref}(\mathbf{W}) =$ _____ (M)

9. Use the answer of #8 to find a basis for the null space of $\mathbf{W} = \mathbf{J}_4 + \mathbf{I}_4$.

The basis will be a set of vectors. Answer: _____

10. Given that $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of \mathbf{J}_4 . Find the corresponding eigenvalue.

$\lambda =$ _____. Hint, just recall $\mathbf{J}_4 * \mathbf{v} = \lambda * \mathbf{v}$.

11. In linear algebra, the **trace** of a square matrix \mathbf{A} , denoted $\text{tr}(\mathbf{A})$, is the sum of the elements on its main diagonal, $a_{11} + a_{22} + \cdots + a_{nn}$. It is only defined for a square matrix ($n \times n$). Use the table below to conjecture a formula for $\text{tr}(\mathbf{J}_n)$, n a positive interger.

n	3	4	5	6	7
$\text{tr}(J_n)$	1	0	1	0	1

Answer: _____

12. Use the table below to conjecture a formula for $\det(\mathbf{J}_n)$, n a positive interger.

n	2	3	4	5	6	7	8	9
det(J_n)	-1	-1	1	1	-1	-1	1	1

Answer: _____

13. Use the table below to conjecture a formula for $tr(\mathbf{W}_n = \mathbf{J}_n + \mathbf{I}_n)$, n a positive interger.

n	3	4	5	6	7	8	9	10
tr(W_n)	4	4	6	6	8	8	10	10

Answer: _____

14. The $det(\mathbf{W}_n = \mathbf{J}_n + \mathbf{I}_n)$ for n a positive interger is zero.

Construct \mathbf{W}_n for $n = 3, 4, 5,$ and 6 . Then conjecture why $det(\mathbf{W}_n) = 0$ for all n .

Answer: _____